DSE-Mech C-scheme 25/01/2023
Sem-III

Duration: 3 hours **Max. Marks: 80**

N.B. (1) Question No. 1 is **COMPULSORY**.
(2) Answer **ANY THREE** questions from Q.2 to Q.6.
(3) Figures to right indicate full marks.

Que. 1 **a.** Find Laplace Transform of $tcos4t.cos7t$ **5**

b. Find Fourier series expansion of $f(x) = x \ in \ (-\pi, \pi)$ **5**

c. Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ **5**

d. If $A=\begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, Find eigen values of A^3-$3A^2$+5A **5**

Que. 2 **a.** Obtain Fourier series expansion for $f(x) = x^2$ in $(0, 2\pi)$ **6**

b. By using partial fractions, find the inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+16)}$ **6**

c. Find the eigenvalues and the eigenvectors of the matrix **8**

$$A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

Que. 3 **a.** Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ **6**

b. Find the Laplace transform of $sinh^5 t$ **6**

c. Using Bender Schmidt method , solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to **8**

$u(0,t) = 0, \ u(1,t) = 0, \ u(x,0) = \sin \pi x \quad 0 \le x \le 1$

Que. 4 **a.** By using Laplace transform, evaluate, $\int_0^\infty \frac{cos3t - cos5t}{t} dt$ **6**

b. Find a , b ,c , d, e if $f(z) = (ax^4 + bx^2y^2 + cy^4 + +dx^2 - 2y^2) + i(4x^3y - ey^3 + 4xy)$ is an analytic function. **6**

c. Obtain the half range cosine series of $f(x) =$ **8**

$$\begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$$

Que. 5 a. Find the analytic function $f(z) = u + iv$, in terms of z, if $u=y^3-3x^2y$ 6

b. If $L\{f(t)\} = \frac{s}{s^2+s+4}$, find $L\{e^{-2t} f(2t)\}$ 6

c. Determine if the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ diagonalizable, hence find it's diagonal matrix D and modal matrix 8

Que. 6 a. Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2-x^2)}{12}$, where $0 < x < \pi$. 6

b. Find inverse Laplace transform of $cot^{-1}\left(\frac{s+3}{2}\right)$ 6

c. Using Crank- Nicholson simplified formula, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, $u(0,t) = 0,\ u(4,t) = 0,\ u(x,0) = \frac{x}{3}(16-x^2)$ for one step for time. 8

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